SOL HW 1.1 By Jennifer Wu & Alice Wang

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Name: Key Date:

M10 Honours: Section 1.1 Quadratic Functions $y = ax^2 + bx + c$

1. Indicate the values of "a" "b" and "c" in each of the following equations:

a) $y = x^2 - 2x - 5$ — $a = 1$ $b = -2$ $c = -5$

b) $y = \frac{1}{2}x^2 + 5$ — $a = \frac{1}{2}$ $b = 0$ $c = 5$

c) $y - x^2 + 2 = 0$ — $y = x^2 - 2$ $a = 1$ $b = 0$ $c = -2$

d) $y = x(x - 7)$ — $a = 1$ $b = -7$ $c = 0$

e) $f(x) = x^2 + 1$ — $a = 1$ $b = 0$ $c = 1$

f) $y = -3(x + \frac{1}{3})^2 - 10$ — $y = -3(x^2 + \frac{2}{3}x + \frac{1}{9}) - 10$ $= -3x^2 - 2x - \frac{1}{3} - 10$ $= -3x^2 - 2x - \frac{31}{3}$ $a = -3$ $b = -2$ $c = -\frac{31}{3}$

2. Factor each of the following quadratic functions and find i) the Coordinates of the Roots, ii) the Equation of the Axis of Symmetry, iii) Coordinates of the Vertex, iv) Domain and Range

a) $y = x^2 + 3x - 18$ — $y = (x - 3)(x + 6)$ Roots: $x = 3, -6$ Axis of S: $x = \frac{3 + (-6)}{2} = -\frac{3}{2}$ Vertex: $y = (\frac{3}{2} - 3)(\frac{3}{2} + 6)$ $y = -\frac{81}{4}$ $(-\frac{3}{2}, -\frac{81}{4})$ Roots: Vertex: A of S: Range: $y \geq -\frac{81}{4}$

b) $y = 2x^2 + 3x - 2$ $a = 2$ $b = 1$ $c = -2$ $x = \frac{-3 \pm \sqrt{9 - (-16)}}{4}$ $= \frac{-3 \pm \sqrt{25}}{4}$ $= \frac{-3 \pm 5}{4}$ $x = \frac{1}{2}, -2$ Axis of S: $x = -\frac{1}{4}$ Vertex: $y = 2(\frac{1}{4})^2 + 3(-\frac{1}{4}) - 2$ $= \frac{1}{8} - \frac{3}{4} - 2$ $= -\frac{25}{8}$ $(-\frac{1}{4}, -\frac{25}{8})$ Roots: Vertex: A of S: Range: $y \geq -\frac{25}{8}$

c) $y = -x^2 - 12x - 35$ $y = -(x^2 + 12x + 35)$ $y = -(x + 5)(x + 7)$ Roots: $x = -5, -7$ Axis of S: $x = \frac{-5 + (-7)}{2} = -6$ $y = -(-6 + 5)(-6 + 7)$ $= -(-1)(1) = 1$ Vertex $(-6, 1)$ Roots: Vertex: A of S: Range: $y \leq 1$

d) $y = x^2 + \frac{5}{2}x - \frac{3}{2}$ $y = \frac{1}{2}(2x^2 + 5x - 3)$ $y = \frac{1}{2}(2x - 1)(x + 3)$ Roots: $x = \frac{1}{2}, -3$ Axis of S: $x = \frac{\frac{1}{2} + (-3)}{2} = -\frac{5}{4}$ Vertex: $y = \frac{1}{2}(2(-\frac{5}{4}) + 1)(-\frac{5}{4} + 3)$ $y = \frac{1}{2}(-\frac{3}{2})(\frac{7}{4})$ $= -\frac{21}{16}$ Roots: Vertex: A of S: Range: $y \geq -\frac{49}{16}$

e) $y = 6x^2 + 13x - 5$ Roots: Vertex: A of S: Range:

f) $y = 15x^2 - 7x - 2$ Roots: Vertex: A of S: Range:

g) $y = 32x^2 - 64x - 27$ Roots: Vertex: A of S: Range:

h) $y = \frac{1}{2}x^2 + \frac{1}{2}x - 6$ Roots: Vertex: A of S: Range:

i) $y = x^2 + \frac{1}{6}x - \frac{1}{6}$ Roots: Vertex: A of S: Range:

	Roots	A of S	Vertex	Range	Domain
d)	$\frac{5}{4}, -\frac{1}{4}$	$x = \frac{5}{8}$	$(\frac{5}{8}, -\frac{169}{16})$	$[-\frac{169}{16}, \infty)$	
e)	$-\frac{3}{8}, \frac{2}{3}$	$x = \frac{7}{48}$	$(\frac{7}{48}, -\frac{529}{96})$	$[-\frac{529}{96}, \infty)$	
h)	$3, -4$	$x = -\frac{1}{2}$	$(-\frac{1}{2}, -\frac{49}{2})$	$[-\frac{49}{2}, \infty)$	

3. Graph the following quadratic functions and label the Roots, Axis of Symmetry, Vertex, and Y-intercepts:

a) $f(x) = x^2 + 7x + 10$ Roots: A of S: Vertex: Y-intercept:

$f(x) = (x + 2)(x + 5)$ Roots $x = -2, -5$ Axis of S: $x = \frac{-2 + (-5)}{2} = -\frac{7}{2}$ Vertex: $y = (-\frac{7}{2} + 2)(-\frac{7}{2} + 5)$ $= (-\frac{3}{2})(-\frac{3}{2})$ $= \frac{9}{4}$ $(-\frac{7}{2}, \frac{9}{4})$ Y-intercept: $y = 0^2 + 7(0) + 10$ $(0, 10)$

b) $f(x) = -2x^2 + 15x + 18$ Roots: A of S: Vertex: Y-intercept:

$-f(x) = (x + 3)(x + 6)$ $x = -3, -6$ Axis of S: $x = \frac{-3 + (-6)}{2} = -\frac{9}{2}$ Vertex: $y = (-\frac{9}{2} + 3)(-\frac{9}{2} + 6)$ $= -\frac{15}{4}$ $(-\frac{9}{2}, -\frac{15}{4})$ $y = (-\frac{3}{2})(\frac{3}{2})$ $y = -\frac{9}{4}$

c) $f(x) = 12 + 5x - 3x^2$ Roots: A of S: Vertex: Y-intercept:

d) $f(x) = -3x^2 - 11x + 6$ Roots: A of S: Vertex: Y-intercept:

e) $f(x) = \frac{15}{2} - 2x - 2x^2$ Roots: A of S: Vertex: Y-intercept:

f) $f(x) = -\frac{8}{3} + 2x + 3x^2$ Roots: A of S: Vertex: Y-intercept:

4. Solve each of the following quadratic equations. Provide your answers in exact form

a) $5x - 1 = 2x^2$ — a) $\frac{5 \pm \sqrt{17}}{4}$

b) $8x + 8 = 12x^2$ — $\frac{1 \pm \sqrt{28}}{3}$

c) $x^2 - 5x + 3 = 0$ — $\frac{5 \pm \sqrt{13}}{2}$

d) $6x + 6 = 15x^2$ — $\frac{1 \pm \sqrt{11}}{5}$

5. Determine the vertex of the parabola $y = 3(x - 20)(x + 22)$

$x = \frac{20 - 22}{2} = -1$

$y = 3(-1 - 20)(-1 + 22) = 3 \cdot -21 \cdot 21 = -1323$

vertex $= (-1, -1323)$

6. A pebble is dropped from a bridge into a river at height "h" meters above. Let "t" be the number of seconds after the release. If $h(t) = 65 - 4.9t^2$, then how high is the pebble after 3 seconds? What is the domain and range of this scenario? When will the pebble hit the ground?

7) After 3s:
$-4.9 \cdot 3^2 + 65 = 20.9$

Domain: $\mathbb{R}$

Range: $0 \le y \le 65$

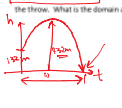
When the pebble hits the ground:
$0 = 65 - 4.9t^2$
$4.9t^2 = 65$
$t = 3.64$

7. A tennis ball is dropped from a balcony. The height of the ball (h) above the ground is given by the formula $h(t) = 78.4 - 4.9t^2$. Where "t" is the number of seconds after release. How high is the balcony from the ground? When will the ball hit the ground?

8) height from balcony to ground
$h(t) = 78.4 - 4.9 \cdot 0^2$
$h(t) = 78.4$

time when ball hits the ground:
$78.4 - 4.9t^2 = 0$
$78.4 = 4.9t^2$
$t = 4$

8. Tom throws a football from the top of his building. The height of the ball is given by the formula: $h(t) = -3t^2 + 60t + 132$ where "h" is the height of the football and "t" is the number of seconds after the throw. What is the domain and range of this scenario? When will the ball be falling to 50m?



$y = -3x^2 + 60x + 132$
$y = -3(x^2 - 20x) + 132$
$y = -3(x^2 - 20x + 100) + 432$
$y = -3(x-10)^2 + 432$

Range: $0 \le h \le 432$

$0 = -3(x-10)^2 + 432$
$144 = (x-10)^2$
$\pm 12 = x - 10$
$10 \pm 12 = x$
$x = 22, -2$

D: $0 \le x \le 22$

9. If the quadratic equation $(x-2)^2 + k = 0$ has two distinct real roots, then what is the range of "k"? (Multiple choice, circle one) Justify your answer.

a) $k > 2$ b) $k < 0$ c) $k \le 0$ d) $k \le 4$

$= x^2 - 4x + 4 + k = 0$

using the discriminant:

$16 - 4(4+k) > 0$
$16 - 16 - 4k > 0$
$0 - 4k > 0$
$4k > 0$
$k > 0$

answer: b

10. Find the values of "A" and "B" if $x^2 + 6x + 16 = (x+A)^2 + B$

$= (x+3)^2 + 16 - 9$
$= (x+3)^2 + 7$

$A = 3 \quad B = 7$

11. Find the values of "A" and "B" if $x^2 - 10x + 27 = (x+A)^2 + B$

$= (x-5)^2 + 27 - 25$
$= (x-5)^2 + 2$

$A = -5 \quad B = 2$

12. The figure below shows the graph of $y = -x^2 + px + q$. The graph cuts the y-axis at point "P". A horizontal line is drawn through points "P" and "Q". What are the coordinates of point "Q"?

$y = -(x - r_1)(x - r_2)$
$y = -(x+1)(x-4)$
$y = -(x^2 - 3x - 4)$
$y = -x^2 + 3x + 4$

$4 = -x^2 + 3x + 4$
$0 = -x^2 + 3x$
$0 = -x(-x+3)$
$x = 0 \quad x = +3$
$y = 4$

P (0,4)
Q (3,4)

13. If the quadratic function $y = ax^2 + bx + c$ has two equal roots and opens up, then which of the following statements are correct?

i) $a > 0$ ii) $c > 0$ iii) $b^2 - 4ac > 0$

14. If $y = (x-2)^2$ and $y = 2x+1$ intersect at points (x_1, y_1) and (x_2, y_2), then which of the following quadratic functions has the roots at x_1 and x_2?

a) $y = x^2 - 6x + 3$ b) $y = x^2 - 2x + 3$ c) $y = x^2 - 6x + 1$ d) $y = x^2 - 2x + 1$

$y = (x-2)^2$
$y = x^2 - 4x + 4$
$y = 2x + 1$

$0 = x^2 - 6x + 3$

answer: a

15. Determine all values of "k" with $k \neq 0$ for which the parabola has its vertex on the x-axis.

$y = kx^2 + (5k+3)x + (6k+5)$

$y = a(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a}$

the vertex is on the x-axis, $y = 0$

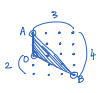
$a(x-p)^2 + q$

$\frac{-(5k+3)^2}{4k} + 6k + 5 = 0$
$25k^2 + 30k + 9 = 24k^2 - 20k$
$k^2 + 50k + 9 = 0$
$k = -25 \pm 2\sqrt{154}$

16. Point "A" is the vertex of the parabola $y = x^2 + 2$, point "B" is the vertex of the parabola $y = x^2 - 6x + 7$, and "O" is the origin. Determine the area of $\triangle AOB$.

A: $y = (x-0)^2 + 2$
vertex = (0, 2)

B: $y = (x-3)^2 - 2$
vertex = (3, -2)



$\Delta AOB = 12 - 6 - 3$
$= 3$

17. Consider the function $f(x) = 2x^2 - 4x + c$. What value of "c" maximizes the product of the roots of the function, given that at least one root is real?

$\frac{4 \pm \sqrt{16 - 8c}}{4}$

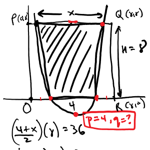
$\sqrt{16 - 8c} \neq 0$

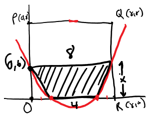
$c = 2$

If:
$16 - 8c \ge 0 \rightarrow$ product of roots is negative
$16 - 8c < 0 \rightarrow$ no real roots
$16 - 8c = 1 \rightarrow (15+1)(15-1) = 15$
$\rightarrow 4 \cdot 4 = 16$

the bigger the difference between the roots, the smaller the product

18. Challenge: square OPQR has vertices O(0,0), P(0,8) , Q(8,8) and R(8,0). The parabola with equation $y = a(x-2)(x-6)$ intersects the sides of the square OPQR at points "K", "L", "M", and "N". Determine all the values of "a" for which the area of the trapezoid KLMN is 36.





$\left(\frac{4+x}{2}\right)(8) = 36$

$(4+x)(4) = 36$

$(4+x) = 9$

$x = 5$

so (6.5, 8) is a coordinate.

(2,0)

$y = a(x-4)^2 + q$

$0 = a(2-4)^2 + q$

$0 = a(4) + q$ ①

$8 = a(6.5-4)^2 + q$

$8 = a(6.25) + q$ ②

$8 = a(6.25) + q$

$0 = a(4) + q$

$8 = (2.25)a$

$8 = \frac{9}{4} = a$

$\frac{32}{9} = a$

$\left(\frac{8+4}{2}\right)(x) = 36$

$6x = 36$

$x = 6$

(2,0)

$0 = 4a + q$

(0,6)

$6 = a(0-4)^2 + q$

$6 = 16a + q$

$0 = 4a + q$

$6 = 12a$

$\frac{1}{2} = a$